**Monty Hall Problem**

The Monty Hall problem involves a contestant trying to win a prize (say a car), which is behind one of several doors in front of the contestant. The contestant is originally given a chance to pick one of the doors. The host (who has knowledge of which door contains the prize) then opens a few other doors (not containing the prize), excluding the one chosen by the contestant. The host now offers a choice for the contestant to pick a new door (that is, switch from the original choice) or to stay (stick with the original choice). The correct strategy would be to switch as it increases the probability of the prize behind the chosen door as compared to the original choice.

A generalized version of this problem with a total of ‘N’ doors and the host opening a total of ‘K’ doors from the other ‘N-1’ doors not containing the prize can be derived.

The probabilities can be understood as:

Given that the initial choice of the contestant, say .

- Say belongs to the universe , and the rest of the doors belong to the universe

The probability of the prize lying behind one of the N doors is .

Thus, the probability of the prize lying behind the door =

Now, the host opens the ‘K’ doors from the universe . This action does not affect the universe , and thus, the probability of the prize being behind the originally chosen door is still P(car in )= .

Note that the two events, the prize being present in the universe and the prize being present in the universe, are mutually exhaustive (since the prize has to be present in either or ).

This brings us to P(car in ) + P(car in ) =1 (as the events are mutually exhaustive).

So, even after the host opens the ‘K’ doors, the above-stated events are still mutually exclusive, and since P(car in ) hasn’t changed, P(car in ) will still remain = 1 - P(car in ) = 1 - .

But note that the universe now contains a total of N-1-K unopened doors. Since the prize has an equal probability of being present in the remaining doors,

P(car in any of the N-1-K unopened doors of the universe ) = P(car in )\* 1 / (N-1-K)

= (N-1)/(N\*(N-1-K))

Since (N-1)/(N\*(N-1-K)) > 1/N for any K > 0, it is obvious to state,

P(car in any of the N-1-K unopened doors of the universe ) > P(car in ) which is,

P(car after shifting from original choice) > P(car sticking to original choice)

So, the reasonable choice for the contestant is to now shift their choice from the originally chosen door.

P(winning the prize after shifting from original choice) = (N-1)/(N\*(N-1-K))

P(winning the prize sticking to original choice) = 1/N

In real life, the above probabilities can be achieved by running the Monty Hall simulator infinite times, proving it is the right choice for a contestant to change/shift their choice once the host reveals the K doors behind which the prize isn’t present.

Consider the example of the original Monty Hall problem, which consists of a total of 3 doors, and once the contestant chose a door, the host then opened one of the two other doors not containing the prize.

The original Monty Hall problem consisting of 3 doors evaluates to a probability of ⅓ of winning the prize, sticking to the original choice, while shifting the choice gives a probability of ⅔ of winning the prize, and thus, the correct way to go is to shift the original choice of the contestant’s door.